A New Approach for the FDTD Modeling of Antennas Over Periodic Structures

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Abstract—A new approach is proposed for the time-domain modeling of antennas over periodic substrates. This class of problems would typically require a time-consuming simulation of the antenna structure with a finite number of unit cells of the periodic substrate, chosen to be large enough to achieve convergence. On the contrary, the present work employs periodic boundary conditions applied at the substrate, to dramatically reduce the computational domain and hence, the cost of such simulations. Emphasis is given on radiation pattern calculation, and the consequences of the truncated computational domain of the proposed method on the computation of the electric and magnetic surface currents invoked in the near-to-far field transformation. The theoretical aspects of the proposed methodology are complemented by numerical examples of a wire and an integrated patch antenna over electromagnetic band-gap substrates.

Index Terms—Antenna radiation patterns, dispersive media, finite-difference time-domain (FDTD) methods, periodic structures.

I. INTRODUCTION

Periodic structures have been widely employed in antenna design as a means of improving the operational characteristics of radiating geometries. In particular, electromagnetic band-gap substrates and high-impedance surfaces have been integrated with well-known planar and wire antennas [1]–[3]. Typically, the design cycle for such geometries includes two steps. First, the periodic structure is independently analyzed and its dispersion characterized. This analysis allows for the specification of the unit cell parameters. Subsequently, the antenna along with a truncated version of the periodic substrate or surface are simulated. The number of unit cells included in the simulation is progressively increased until the convergence of the electromagnetic response of the combined antenna/periodic structure geometry. While the first step is usually fast and involves the simulation of a single unit cell terminated in periodic boundary conditions, the second step can be computationally expensive as it involves multiple unit cells, and the antenna itself.

The complexity gap between the aforementioned modeling stages partly stems from the fact that the combined structure is non-periodic, hence not compatible with periodic boundary conditions that could substantially reduce the associated computational domain. The main purpose of this communication is to bridge this gap, within the context of the finite-difference time-domain (FDTD) method. Our approach extends recent work aimed at the modeling of microstrip-based microwave circuit geometries on periodic substrates [4], [5] to the case of integrated or wire antenna interaction with infinite periodic structures. It is shown that this interaction can be fully captured by a reduced domain, where periodic boundary conditions are employed around the periodic structure under consideration, with absorbing boundaries terminating the open boundaries of the domain above the substrate. While some preliminary results have been reported in [6], a detailed presentation of the proposed methodology is given here. Moreover, the unconventional form of the proposed computational domain requires the modification of the well-known near to far-field transformation for the determination of antenna radiation patterns via FDTD. Such a modification, accelerated by means of periodic boundary conditions, is outlined and validated.

Two applications of this new methodology, namely a parallel monopole and an integrated patch antenna over electromagnetic bandgap substrates, provide a clear demonstration of its accuracy and efficiency, compared to the standard FDTD alternative.

II. METHODOLOGY

A. The Array-Scanning Sine-Cosine FDTD Method With Composite Periodic/Absorbing Boundaries

In this section, a brief overview of the application of the sine-cosine FDTD [7], augmented with the array-scanning technique [8], to problems involving non-periodic structures and sources over periodic substrates is presented. An example of such a case is the patch antenna over a two-dimensional periodic substrate excited by a broadband voltage source, shown in Fig. 1. Notably, the presence of the patch as well as the source render the whole problem non-periodic, hence prohibiting the direct use of periodic boundary conditions and the reduction of the computational domain to a single unit cell of the substrate. The isolated source can be modeled within one unit cell of the periodic substrate by applying the sine-cosine array-scan FDTD. Let \( \mathbf{r}_0 \) be a point within the unit cell and \( \mathbf{E}_{array}(\mathbf{r}_0, \mathbf{e}, t) \) the electric field over time determined by the sine-cosine method, for Bloch wave-vectors \( \mathbf{e} \), with \(-\pi/\delta_x \leq k_x \leq \pi/\delta_x \) and \(-\pi/\delta_y \leq k_y \leq \pi/\delta_y \), where \( \delta_x \), \( \delta_y \) are the periodicities in the \( x, y \)-direction respectively. The electric field \( \mathbf{E}_0 \) either inside or outside the computational domain due to the original source alone can be found by

\[
\mathbf{E}_0(\mathbf{r}_0 + \mathbf{r}_{i,1}, t) \approx \frac{1}{N^2 M^2} \sum_{n = -N/2}^{N/2} \sum_{m = -M/2}^{M/2} \mathbf{E}_{array}(\mathbf{r}_0, \mathbf{e}, t) \exp(-j \mathbf{e}(n,m) \cdot \mathbf{r}_{i,1})
\]

with \( \mathbf{e}(n,m) = (2\pi n/\delta_x) + (2\pi m/\delta_y), \mathbf{r}_{i,1} = \hat{x}d_x + \hat{y}d_y \), for integer \( i, j \).

Meanwhile, the metallic surface of the antenna is modeled by setting the tangential electric fields to zero. As a result, periodic boundary conditions reproduce these tangential field zeros from \(-\infty\) to \(+\infty\) along the directions of periodicity, as shown in Fig. 1(a). This problem can be effectively resolved by using a composite periodic/absorbing boundary [4], [5], which confines the application of periodic boundary conditions within the substrate and employs a perfectly matched layer absorber (PML) onwards. Thus, the combination of the array-scanning and the sine-cosine method is applied to simulate the actual structure, as shown in Fig. 1(b).

Convergence of the field expressions in (1) depends on the numbers \( N, M \) of the discrete wave-vectors sampled in each direction of periodicity. However, these simulations are totally independent from each other and perfectly parallelizable as such. Hence, in a parallel environment, the execution time of this method does not change with the
number of simulated wavenumbers, as long as that number does not exceed the nodes available. This is the case for the simulations reported in this communication, which were performed on a Linux cluster. Thus, this technique presents a readily realizable paradigm for FDTD parallelization of quasi-periodic problems, based on spectral rather than spatial domain decomposition.

**B. The Array-Scanning Sine-Cosine FDTD Applied to Antennas Over Periodic Structures: Radiation Pattern Calculation**

In FDTD-based antenna simulations, a near-field to far-field transformation is typically employed for the extraction of radiation patterns. This transformation is based on the equivalence principle, whereby equivalent surface electric and magnetic current densities $J_s = \hat{n} \times \mathbf{E}$, $M_s = \mathbf{H} \times \hat{n}$ can be used to calculate the radiated fields of the antenna-periodic structure system as a whole. For simplicity, let us consider the case where radiated fields in the half-space above the antenna are sought for. To that end, a planar surface right above the antenna is chosen, with $\hat{n} \equiv \hat{z}$ (Fig. 2). Fields radiated in the half-space bounded by this infinite surface are found by evaluating the appropriate radiation integrals [9] and thus the radiation pattern of the antenna is also computed. Practically, the confinement of $J_s, M_s$ over a finite portion of the surface enables the truncation of the infinite radiation integrals.

As a consequence of limiting the computational domain, the equivalent current surface may practically have to extend beyond the boundaries of the domain, to allow for a sufficient decay of the surface currents, which in turn enables the truncation of the radiation integrals. Hence, the values of $J_s, M_s$ on the surface cannot be computed directly everywhere. Instead, an indirect, yet straightforward methodology has been devised. Note that within the periodic structure, (1) provides for the calculation of field values for $-\infty < x, y < +\infty$. Hence, if the boundary of the periodic structure and the space above it is at $z = z_s$, $J_s = \hat{z} \times \mathbf{E}, M_s = \mathbf{H} \times \hat{z}$ can be found at $z = z_s^+$ for all $x, y$. Then, letting the equivalent surface be as close to the interface as the antenna allows (mathematically, at $z = z_s^+$), $J_s, M_s, x = z_s^+$ can be fairly accurately determined by extrapolation, since tangential fields remain continuous across the interface. Note that this process is not applied to metallic antenna surfaces need to always be within the computational domain.

To express this idea in mathematical form, consider the computation of the equivalent surface electric current at a point $\mathbf{r}_0(i\Delta x + M d_x, l \Delta y + N d_y, q_s \Delta z)$ outside the computational domain, with $\Delta x, \Delta y, \Delta z$ being the Yee cell dimensions, $d_x$ and $d_y$ the periods of the two-dimensional periodic structure, while $q_s = z_s / \Delta z$ is the $z$-index of the Yee cells with tangential magnetic field components on the surface. If the computational domain in the substrate includes $N_x \times N_y$ cells, $i \leq N_x, l \leq N_y$. Finally, let cells $(i, l, q)$ with $0 \leq q \leq q_{n_{max}}$ belong to the substrate, where $q_{n_{max}} < q_s$, yet the difference $(q_s - q_{n_{max}}) \Delta z = \delta$ is electrically small (typically 1–2 cells).

Under these assumptions, $\mathbf{P}_E(\mathbf{r}_0)$ can be determined in two steps. First, it can be expressed as an extrapolation function of $\mathbf{P}_E$ inside the substrate, yet still outside the computational domain

$$\mathbf{P}_E(\mathbf{r}_0) = f \left( \mathbf{P}_E(\mathbf{r}_0 - \delta \hat{z}), \mathbf{P}_E(\mathbf{r}_0 - (\delta + \Delta z) \hat{z}) \right), \quad (2)$$

where $f$ is the zeroth order or higher order extrapolation function. It has been numerically observed that the choice of the extrapolation order has very little impact on the accuracy of the response. Thus, the direct zeroth-order interpolation is applied in the following examples. A similar methodology can be used to determine the tangential electric fields $\mathbf{E}_T$ required for the computation of the surface magnetic current density $\mathbf{M}_s$ on the surface $z = q_s \Delta z$.

Second, the values of $\mathbf{P}_E$ included in these extrapolations can be computed from nodes that do belong to the computational domain through the Floquet boundary conditions. Indeed, for their phasors

$$\mathbf{P}_E(i\Delta x + M d_x, l \Delta y + N d_y, q_s \Delta z) = \mathbf{P}_E(i\Delta x, l \Delta y, q_s \Delta z) \exp (-j(k_x M d_x + k_y N d_y)). \quad (3)$$

Finally, the FDTD implementation of the latter through the sine-cosine method provides the time-domain values of these components at each node in the proposed methodology as follows.

Microstrip-fed patch antennas (Fig. 3(a)) are necessarily treated as one-dimensional periodic structures in the transverse direction. Along the direction of the microstrip, the computational domain is terminated
in absorbing boundary conditions, which allow for the calculation of the antenna input impedance. As long as the microstrip is either at the back or on top of the substrate, hence at a plane terminated at absorbing boundary conditions, the use of periodic boundary conditions for the termination of the substrate in the transverse direction is possible.

A more complicated case arises in the study of antennas fed with coaxial cables (Fig. 3(b)). In this case, the presence of the coaxial cable inside the periodic substrate cancels the periodicity of the substrate and hence, prohibits the application of periodic boundary conditions. To overcome this problem, the following approximate solution has been devised. The coaxial cable is modeled by a one-dimensional transmission line separately from the main computational domain, and is connected to the feed point of the slot antenna using the thin-wire feed model introduced in [10]. Since the feed point is above the periodic substrate, the excitation is applied to the antenna, while the substrate periodicity remains intact.

The aforementioned approach does not account for the interaction between the antenna and the feed itself (for example, scattering of near-field waves from the metallic feed boundaries), as the feed has been removed from the mesh. However, the results of the next section demonstrate that it can still lead to the accurate computation of the input impedance and the radiation pattern of coaxial-fed structures. On the other hand, this methodology would not be applicable in the study of the feed itself (for example, the optimization of its geometry).

III. NUMERICAL RESULTS

A. Horizontal Monopole Over a High Impedance Surface

The first example, used as a benchmark application, is a horizontal monopole antenna, mounted on a high-impedance ground plane (a two-dimensional periodic “mushroom surface”) that is shown in Fig. 4 [3]. The 2.4 cm × 2.4 cm unit cell of the high-impedance surface consists of a square metallic plate and a metallic via of a 0.36 mm diameter. The gap between neighboring plates is 0.15 mm. The unit cell of the mushroom structure resides in a 1.6 mm thick homogeneous substrate. The 0.1 mm diameter monopole is modeled by the thin wire model [11], and the coaxial feed is modeled by a one-dimensional transmission-line with 100 cells. The transmission-line model is connected to the input of the antenna at point A (Fig. 4) above the periodic surface. The computational domain at and above the horizontal plane where the antenna is located is terminated by 10-cell PMLs, while periodic boundary conditions are employed below the plane, in both the x- and y-directions. A modulated 8–18 GHz Gaussian pulse is applied at the 50-th cell of the transmission-line. The time step is set to 0.88 ps and 25000 time steps are executed.

A computational domain of 2 × 6 unit cells in the x- and y-directions respectively is used, in order to enclose the wire antenna. Uniform sampling of 16 k_x points and 16 k_y points within the irreducible Brillouin zone of the high-impedance surface is applied, resulting in a total of 256 samples of the wave-vectors. For comparison, a finite structure, 3 cm × 3 cm in the x- and y-directions respectively, is also simulated. The same finite structure is simulated and measured in [3].

Fig. 5 shows the reflection coefficient (S_{11}) at the input of the wire antenna, computed by the proposed method and the finite-structure simulation, along with measured results of [3]. The agreement between the four sets of data is excellent. The array-scanning based simulation takes 855 seconds per wavevector, while the finite structure simulation needs 6587 seconds.

To compute the far-field pattern, surface currents are recorded on a 14.4 mm × 14.4 mm planar surface right above the wire antenna, with zeroth order extrapolation. Fig. 6 depicts the E-plane pattern at 13 GHz using FDTD (proposed method and finite structure simulation) along with the measured results of [3], corroborating the excellent agreement observed in the S_{11} results of Fig. 5. Results are shown for the half-space above the substrate only, as an infinite periodic substrate,
Fig. 6. The E-plane pattern of the horizontal monopole at 13 GHz using the proposed method and the finite structure simulation, compared with the measured results of [3], for the half-space above the substrate.

Fig. 7. (a) The patch antenna on an electromagnetic band-gap substrate of [2] and (b) the unit cell of the substrate.

The next example is a patch antenna printed on an electromagnetic band-gap substrate, which was presented in [2]. The geometry of the structure is shown in Fig. 7. The substrate includes two layers with thicknesses \( h_1 = 0.787 \text{ mm} \) and \( h_2 = 0.787 \text{ mm} \) and a dielectric constant \( \varepsilon_r = 4.6 \). The bottom layer of the substrate consists of an array of mushroom structures with a unit cell size of 7 mm. The size of the metallic patch is \( 4.5 \text{ mm} \times 4.5 \text{ mm} \), and the radius of the via is 0.2 mm. The patch antenna is printed on the surface of the upper substrate layer, with dimensions \( L = 22.14 \text{ mm} \) and \( W = 31.59 \text{ mm} \). The antenna is excited by a coupled microstrip line of 1.2 mm width along the \( y \)-direction, printed on the interface of the two substrate layers and aligned with the center of the patch.

In FDTD, each unit cell is modeled by 36 \( \times \) 36 \( \times \) 32 Yee’s cells, and the thickness of each layer of the substrate is represented by 6 Yee’s cells. A modulated 2–4 GHz Gaussian pulse is applied as the excitation. The time step is set to be 0.77 ps, and 32768 steps are performed. The computational domain is set to 6 unit cells in the \( x \)-direction and 4 unit cells in the \( y \)-direction. In the \( x \)-direction, the computational domain is terminated by periodic boundary conditions within the lower layer of the substrate, and by 10-cell PMLs elsewhere. In the \( y \)-direction, PMLs are applied at both ends of the domain. In the \( x \)-direction, 32 \( k_x \) points are sampled in the Brillouin zone. A finite structure with 6 \( \times \) 8 unit cells is also simulated for comparison.

Fig. 8 shows the \( S_{11} \) obtained using the proposed method and the finite structure simulation, along with the measured results of [2], again for the half-space above the substrate. Excellent agreement is demonstrated for the results of the array-scanning based analysis. The execution time for the sine-cosine array-scanning FDTD was 2351 seconds per wavevector. The finite structure simulation took 22478 seconds.

Furthermore, the radiation pattern is computed, by applying a near to far-field transformation on a surface of 10 \( \times \) 8 unit cells in the \( x \)- and \( y \)-directions and symmetrically placed with respect to the proposed computational domain and half a Yee’s cell above the patch antenna. Fig. 9 shows the E-plane far-field pattern of the patch antenna at 2.5 GHz using the proposed method, compared with finite simulation results and the measured results of [2], for the half-space above the substrate.
Efficient Incorporation of a PEC/PMC Plane in the Multiple-Grid Adaptive Integral Method

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Abstract—The multiple-grid adaptive integral method (MG-AIM) is extended for fast analysis of scattering from piecewise homogeneous structures on or above a perfect electrically/magnetically conducting plane. Because the Green functions in the relevant integral equations contain (i) reflection terms that are in correlation form in the direction normal to the plane and (ii) direct terms that are in convolution form in all directions, the MG-AIM propagation stage gives rise to Hankel-(two level) block-Toeplitz matrices in addition to the usual (three level) block-Toeplitz matrices. These additional matrices are multiplied with trial vectors during the iterative solution stage by using FFTs; however, to improve efficiency, the FFTs computed for multiplying the direct terms are recycled. Numerical examples show that the proposed method requires (almost) half the number of computations and storage space compared to a brute-force imaging scheme for structures that terminate on the plane; larger gains are observed for structures residing above the plane.

Index Terms—FFT, image theory, integral equations.

I. INTRODUCTION

Perfect electrically/magnetically conducting (PEC/PMC) planes are used in many scattering applications to reduce the analysis complexity and to simplify/emphasize part of the wave physics, e.g., the earth is often modeled as a PEC plane as a first approximation when characterizing communication channels in forests [1]. When the application of interest involves piecewise homogeneous structures, surface integral equation based simulators accelerated by fast algorithms enable efficient analysis [2], [3]; such simulators can model a PEC/PMC plane using two approaches based on the method of images: The “brute-force imaging” approach removes the plane, introduces the image of the structure of interest (and the excitation), and finds unknown currents on the actual structure and its image. The “Green-function modification” approach adds appropriate reflection terms to the homogeneous-medium Green functions (and to the excitation) and finds unknown currents on the actual structure. Brute-force imaging is more straightforward because it does not require any changes to an existing simulator (only a pre-processing step to image the structure mesh is needed); yet, it is less efficient: It doubles the number of homogeneous regions not terminated on the plane and it doubles the volume and the bounding-surface area of regions terminated on the plane; thus, it produces (almost) twice as many unknowns as Green-function modification. (The ratio is strictly less than two when the plane terminates regions because of junction treatment, see Section II-B). As a result, a classical iterative method of moments (MOM) solver using Green-function modification would require (almost) half of the memory space, half of the matrix-fill time, and half of the matrix-solve time per iteration for general multi-region problems. For important special cases, e.g., two-region problems, it can even require (almost) a quarter of the memory space and matrix-solve time per iteration compared to brute-force imaging (Section II-A). Moreover, iterative MOM solvers using Green-function modification generally require smaller numbers of iterations for