Metallic Transmission Screen for Sub-wavelength Focusing

A.M.H. Wong, C.D. Sarris and G.V. Eleftheriades

Abstract: A simple metallic transmission screen is proposed that is capable of focusing an incident plane wave into a sub-wavelength spot in the near field. The principle of operation is inspired by holographic concepts applied to the near-field. A corresponding design procedure is described and supporting full-wave simulation results are provided.

Introduction: Strong recent interest has centered on near-field imaging systems which produce sub-wavelength resolution. These imaging systems promise increased capabilities in microscopy, lithography, and near-field sensing amongst other applications [1-3]. This paper describes a simple yet novel metallic transmission screen which focuses an incident electromagnetic wave to a sub-wavelength spot in the near field. The transmission screen was inspired by concepts from holography, where a record of the interference between two waves is used to convert a reference waveform into the desired object waveform [4]. In the remaining sections of this paper, we will explain our two-stage design process, in which we first design an appropriate (in general complex) transmission function, and then implement it with a binary metallic screen. We will also display corresponding results from a scalar diffraction calculation and a full-wave simulation demonstrating sub-wavelength resolution.

Design of Transmission Function: Fig. 1 shows a diagram depicting our problem of transmission function design. We use, as the reference wave, a normally incident plane wave described by

\[ E_{\text{ref}}(x, z) = \exp(-ik_0z) \]

and desire to reconstruct a converging wave described by,
\[ E_{\text{con}}(x,y) = \frac{\exp(ikr)}{r} \text{ where } r = \sqrt{x^2 + (z-d)^2} \] (1)

and \( d \) represents the focal distance. In operation, the transmission function will be multiplied with the reference waveform to form a new waveform \( E_{\text{rec}} \) immediately to the right of the interference plane, which will reconstruct \( E_{\text{con}} \) at the image plane. While the choice of the field distribution at the image plane is in general arbitrary, we have been motivated to reconstruct \( E_{\text{con}} \) in the form of (1) because earlier work on a negative-refractive-index slab lens \([5]\) has revealed similar field variation close and to the right of the image plane (i.e. away from the lens). Furthermore, the near-field spectrum of (1) has ample evanescent-wave components (see Fig. 2), thus allowing image formation with sub-wavelength resolution (other suitable evanescent-wave “rich” distributions would include \( \exp(jkr)/r^2 \) etc).

We now outline an approach to designing a general transmission function \( T(x) \), which converts an arbitrary electric field \( E_{\text{ref}} \) into a chosen \( E_{\text{con}} \) at the image plane. Although we have, for simplicity, chosen to assume 2D propagation (i.e. \( \partial/\partial y = 0 \) in Fig. 1), the following procedure can be naturally extended to a 3D scenario. We seek to determine, through the principle of back-propagation, the desired waveform \( E_{\text{rec}} \) at the interference plane which will lead to the waveform \( E_{\text{con}} \) at the image plane. First, we expand our desired converging wave (1) as a spectrum of propagating and evanescent plane waves along an auxiliary plane, a short distance \( s \) to the right of the image plane (see Fig.1).

\[ S(k, z = d + s) = \int_{-\infty}^{+\infty} E_{\text{con}}(x, z = d + s) \exp(-ik_x x) dx = K_0 \left( is\sqrt{k_o^2 - k_x^2} \right) \] (2)

, where \( K_0 \) is the zero order modified Bessel function of the second kind. Note that the image plane
(z=d) was avoided due to the singularity caused by the implied presence of a source by (1), which cannot be physically reproduced by the superposition of plane waves. Moreover, all evanescent waves decay in the +z direction, hence evanescent components are enhanced as they are back-propagated from the auxiliary plane to the interference plane; this enhancement of evanescent components is crucial to obtaining sub-wavelength focusing capability. We may now express the spectrum at the interference plane as,

\[ S(k_z, z = 0) = K_0 \left( i s k_z \right) \exp \left( -i k_z (s + d) \right), \text{ where } k_z = \sqrt{k_0^2 - k_x^2} \]  

Finally, we perform an inverse Fourier transform to obtain \( E_{\text{rec}} \). After obtaining \( E_{\text{rec}} \), we can easily find the required transmission function of the screen by

\[ T(x) = E_{\text{rec}}(x)/E_{\text{ref}}(x, z = 0) = E_{\text{rec}}(x) \]  

\( E_{\text{ref}} = 1 \) for a normally incident plane wave. We follow the above procedure to design a transmission function which converts a normally incident plane wave into a waveform focusing at a distance of \( d = \lambda/10 \). To handle the diverging nature of the spectrum (3) we have truncated it to a bandwidth of \([-5k_0, 5k_0]\). This implies that the actual spectrum to be reconstructed (2) is truncated to a maximum wavenumber \(|k_{xm}| = 5k_0\). Since this bandwidth is 5 times the bandwidth of conventional imaging systems which transmit only propagating waves, we expect the optimal focusing quality of the resulting screen to be about \( \lambda/10 \) — a five-fold improvement over the diffraction limit. Fig. 2 shows the spectra at the auxiliary plane (solid) and the interference plane (dash-dot). A growth in the evanescent components can be clearly observed at the interference plane. The inset of Fig. 3 shows the real part of the designed transmission function. This function has a (sub-wavelength) periodicity which matches that of the bandwidth limit \( k_{xm} \); this is essential for encoding the sub-wavelength information onto the screen. Fig. 3 shows the corresponding field distribution along the image plane of which the full width at half maximum (FWHM) is 0.13\( \lambda \). This is indeed roughly a five-fold improvement over the
FWHM of $0.6\lambda$ for a diffraction limited sinc(x) function. We note that higher resolution can be obtained by simply using a wider bandwidth $|k_x|$ thus resulting to a correspondingly narrower focal width. However this is associated with an attenuation in magnitude from the screen to the image plane on the order of $\exp(-k_xd)$.

**Metallic Screen Implementation of Transmission Function:** Using a metallic screen, we have implemented a close approximation to our transmission function, following a technique inspired by earlier work on far-field microwave holographic antennas [6]. In holography, the transmission function is the intensity of the interference pattern of the object and reference waveforms:

$$ T_{\text{approx}}(x) = \| E_{\text{ref}}(x, z = 0) + E_{\text{rec}}(x) \|^2 = 1 + 2 \text{Re}\{E_{\text{rec}}(x)\} + \| E_{\text{rec}}(x) \|^2 $$

Upon reconstruction, one retrieves the image $E_{\text{rec}}$, and the conjugate image $E_{\text{rec}}^*$ and two other terms contributing to zero-order transmission. Since these zero-order transmission terms only contribute to background noise, we suppress them to obtain $T'_{\text{approx}}(x) = \text{Re}\{E_{\text{rec}}(x)\}$. To realize this function with metallic strips and slits, we follow [6] in placing metal strips to represent the transmission function’s periodicity and fringe width. However, while [6] padded positive (high intensity) fringes with metallic strips, we pad the negative cycles of $T'_{\text{approx}}(x)$ with metallic strips. We also slightly alter the sizes of metallic strips such that the field amplitudes transmitted by the sidelobes are in proportional agreement with those described in $T'_{\text{approx}}(x)$. Fig. 1 shows a diagram of the transmission screen, and Table 1 summarizes the locations and widths of the slits on the metallic screen. Full-wave simulations with COMSOL at 3GHz show a FWHM of $0.25\lambda$ at a distance $0.1\lambda$ away from the metallic screen. The corresponding magnitude of the electric field at this image plane is shown in Fig. 4. As shown, the focusing quality of the metallic screen does not fully match that of the complex screen. It can be shown
that this mismatch can be mainly attributed to the inability of a binary screen to produce both positive and negative field values (see inset Fig. 3). This produces a non-zero “DC” distribution around \( k_x = 0 \) which superimposes on the desired distribution at the image plane of Fig. 3 and produces the observed beam widening in Fig. 4 near the null-points. Nevertheless, the obtained FWHM of 0.25\( \lambda \) still represents a dramatic improvement over the diffraction-limited FWHM of 0.6\( \lambda \).

**Conclusion:** Through holographic principles applied to the near-field, we have prescribed a method for implementing simple metallic transmission screens, capable of focusing incident electromagnetic waves into sub-wavelength spots in the near-field. It should be noted that while preparing this manuscript, a recent paper [7] came to our attention describing related near-field focusing structures. However the formulation in [7] is quite different and is not based on the principle of holography; in fact [7] only accounts for the focusing of evanescent waves. Moreover, [7] merely proposes specific screen current distributions but not any specific physical structures that produce these required currents. Finally the structures of [7] require positive and negative index regions. In contrast, we hereby propose readily realizable holographic screen patterns that only require simple slits cut on a ground plane.

**References**


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Figure Captions:

Figure 1: a. A diagram depicting the setup of the transmission screen. Fig. 1: b. A metallic transmission screen designed for focusing a 3GHz incident plane wave to a sub-wavelength spot at the image plane.

Figure 2: a. Comparing the normalized spectrum at the auxiliary plane $s = 0.001\lambda$ (solid) and a plane $0.5\lambda$ to the right of the image plane (dot) shows that $E_{con}(x,y) = \exp(ikr)/r$ contains significant evanescent components in the near-field. Fig. 2b. A comparison of the spectra at the auxiliary plane (solid) and the interference plane (dash-dot).

Figure 3. Inset: The design transmission function. Main: The field distribution at the image plane, after applying the transmission function to a reference wave $E_{ref}$, with a field amplitude 1 V/m. The real part of the field, which comprises more than 98% of the electric field energy, is shown in the figure. The focal width at half maximum is $0.13\lambda$. 
Figure 4: Full-wave field distribution at the image plane (normalized), after shining a normally incident 3GHz plane wave $E_{ref}$ onto the designed metallic screen. The focal width at half maximum is $0.25\lambda$, as opposed to a diffraction limited pattern of $0.6\lambda$. For this simulation, the screen was embedded in a larger ground plane to avoid edge diffracted effects.

Figure 1

![Diagram](image-url)
Figure 2

(a) Normalized Spectrum

(b) Spectrum

\[ \frac{k_x}{k_0} \]
Figure 3
Figure 4
Table 1

<table>
<thead>
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<th>Width</th>
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<td>$0 \lambda / 0\text{mm}$</td>
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<td>$0.062 \lambda / 6.2\text{mm}$</td>
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<tr>
<td>$\pm0.450 \lambda / \pm45.0\text{mm}$</td>
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<td>$0.075 \lambda / 7.5\text{mm}$</td>
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Table 1. The location and sizes of slits on the metallic screen (see Fig. 1).