Finite-Difference Time-Domain Analysis of Metamaterial-Based Leaky-Wave Antennas

Titos Kokkinos*, Costas D. Sarris, and George V. Eleftheriades
The Edward S. Rogers Sr. Department of Electrical and Computer Engineering
University of Toronto, Toronto, ON, M5S 3G4, Canada

Abstract

In this paper, a novel technique for the efficient modeling of 2D Negative-Refractive-Index (NRI) metamaterial leaky-wave antennas (LWAs) is proposed. Specifically, the array factor method is combined with full-wave Finite-Difference Time-Domain (FDTD) simulations of a single unit cell of the NRI metamaterial structure in order to calculate the radiation patterns of these LWAs.

Introduction

Leaky-wave antennas have attracted profound interest for many decades because of their unique characteristics and their comparative advantages (narrow beam, frequency scanning, non resonant). The interest for this type of antennas has been recently renewed because of the realization of Negative-Refractive-Index (NRI) metamaterials and the possible use of those materials for the construction of LWAs which will radiate in the backward direction. Such structures have already been reported in the literature [1]-[3]. The backward radiated beams of these LWAs can be scanned with frequency over a great number of angles, while both fan as well as pencil beams can be produced using 1D or 2D structures [4].

On the front of modeling, the radiation properties of these LWAs have been modeled mostly using transmission-line based models and representing their radiation characteristics with a radiation resistance, or by brute-force simulating numerically the entire structure. The first technique is computationally efficient but can provide only approximate results, while the second is computationally inefficient. In this paper, a rigorous, yet computationally efficient, technique will be presented and applied to the modeling of leaky-wave radiation from a 2D loaded transmission-line-based NRI medium [5]. In particular, a recently introduced algorithm that extends the well-known sine-cosine method of periodic structure analysis to leaky-wave geometries is employed for the calculation of the complex propagation constant in a unit cell of the NRI medium of [5]. Subsequently, the phased array approach of [6] and [7] is used for the extraction of the radiation characteristics of the medium, including gain and patterns.

2D Leaky-Wave Antennas Modeled as Phased Arrays

The periodic 2D structure under consideration is shown in Fig.1. This LWA consists of an infinite number of NRI metamaterial cells, similar to those presented in [5], periodically repeated in space. The unit cell dimension \( d \) is much smaller than the
operating wavelength and therefore the 2D structure can be treated as an effective aperture. Moreover, it is assumed that the structure is excited with an array of sources placed at the leftmost edge and being parallel to the y-axis. As a result, a traveling wave along the x-axis will be supported having a wavefront along the y-axis. Because of the one-dimensional propagation of the waves and the effective

aperture condition \( d \ll \lambda \), the 2D LWA can be modeled as a phased array antenna ([6], Eq. 7), with the axis of the array being parallel to the axis of the wave propagation. This is shown in Fig. 2. The number of the elements of the array is equal to the number of the NRI metamaterial unit cells along the x-axis and the distance between the array elements is equal to the dimension of each unit cell. The fact that this array emulates a LWA is taken into account in the definition of the excitations of the array elements. More specifically, it is well-known that any leaky-wave structure is characterized by a complex propagation constant \( \gamma = \beta - j\alpha \) and therefore a wave traveling along the x-axis has a propagation factor of the form \( e^{-j\gamma x} = e^{-\alpha x}e^{-j\beta x} \). As a result, the array would properly model the leaky-wave structure if its elements are excited with currents of the form \( I_n = I_0 e^{-n\alpha d}e^{-j\varphi_n} \), where \( \varphi_n \) is always a product of the per unit cell phase shift, e.g. \( \varphi_n = n\beta d \). The normalized array factor of this equivalent array is given in Eq. 1. Under the condition of the effective aperture, each element of this array could be assumed to be an isotropic point radiator and therefore Eq. 1 can be also used for the calculation of the radiation patterns of the LWA.

\[
AF = \sum_{n=0}^{N-1} e^{-n\alpha d}e^{jn(k_o d \cos \theta - \beta d)}
\]  

(1)

In order for the array factor analysis to be used for the extraction of the radiation patterns of the metamaterial LWA, the unknown parameters \( \alpha \) and \( \beta \) of Eq. 1 should be calculated. In this study, the use of the Finite-Difference Time-Domain technique, combined with Periodic Boundary Conditions, is suggested for the accurate estimation of these parameters. As presented in [8], the FDTD technique can be applied for the full-wave simulation of a single unit cell of the structure and the inclusive characterization of it. Furthermore, the extension of this methodology, presented in [9], can be used for the simulation of any periodic leaky-wave structure and the calculation of its complex propagation constant. Using this analysis, it is possible to calculate the parameters \( \alpha \) and \( \beta \), through the full-wave simulation of a single unit cell of the NRI metamaterial. This is achieved in an efficient way in
terms of execution time and the consumed computational power. Eventually, these results are combined with Eq. 1 and the accurate radiation patterns of any periodic LWA, which can be treated as an effective aperture \((d << \lambda)\), are produced.

Results

In this section, the radiation patterns of the backward radiation region of the 2D LWA of Fig. 1 will be presented. Although Eq. 1 converges for \(N \rightarrow \infty\), in the following results \(N\) is given a specific value, \(N = 40\), in order to simulate a more realistic structure and compare the numerical results with experimental results. The dimensions of the NRI metamaterial unit cell are \(d_x = d_y = 5\, \text{mm}\), the height of the substrate is \(h = 1.524\, \text{mm}\), the dielectric permittivity of the substrate is \(\varepsilon_r = 2.94\) and the characteristic impedance of the hosting transmission-line segments is \(Z_0 = 126.4\, \Omega\). The values of the lumped elements are assumed to be \(L = 5.6\, \text{nH}\) and \(C = 1\, \text{pF}\). These values of the structure parameters result in a dispersion diagram with a stop-band (band gap) in the case where the phase shift per unit cell is 0 \((\beta d = 0)\) (Fig. 3) [4]. Referring to the dispersion diagram of Fig. 3, it is well-known that the modes which will result in leaky-wave radiation are those lying inside the light cone \((\beta < k_o)\) As far as the backward radiation region is concerned, this is explicitly shown in Fig. 3. The E-plane patterns of the LWA produced by the excitation of the modes of this band are shown in Fig. 4. According to these patterns, the antenna starts to leak radiation at \(-35^\circ\) (2.688 GHz), while the gain increases toward broadside, reaching its maximum value at \(-11^\circ\) (2.78 GHz). However, the gain starts decreasing just before broadside, reaching its minimum value at broadside. These results agree with the experimental radiation patterns of [4]. In Fig. 5 the values of \(\alpha\), together with the corresponding scanning angles as a function of frequency are presented.

Conclusion

The FDTD analysis of an NRI-based LWA structure was presented in this paper. This analysis was accomplished by means of a recently introduced extension of the
sine-cosine method of periodic boundary condition implementation, that enabled the calculation of leaky-wave complex propagation constants, by simulating a unit cell of the structure. Numerical results indicated the ability of the method to accurately model the backward-wave radiation characteristics of a 2D-NRI LWA. An earlier experimental characterization of the latter corroborated the proposed analysis approach of this work.

References