Accelerated Time-Domain Modeling of Microstrip Based Microwave Circuit Geometries on Periodic Substrates

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Abstract—Advances in the design of electromagnetic band-gap and meta-material structures have opened new possibilities for substrate engineering to optimize the performance of microstrip or coplanar waveguide based integrated microwave circuits. Periodic substrates alone can be fully characterized through their dispersion analysis. On the other hand, modeling the interaction between metallic guides and such substrates usually involves lengthy simulations, because the resulting geometry is non-periodic. We propose a Finite-Difference Time-Domain based solution to this problem, which leads to significant acceleration of the relevant modeling process over broad bandwidths, without compromising accuracy.

Index Terms—FDTD, periodic boundary conditions, microwave circuits.

I. INTRODUCTION

PERIODIC structures have been an old, yet always timely, research subject [1], [2]. In particular, the use of periodic substrates either in the form of artificial dielectrics or electromagnetic band-gaps has enabled the design of planar microwave circuits with enhanced functionality [3], [4] and unique properties [5]. From a modeling point of view, the dispersion analysis of periodic structures can be usually carried out in an efficient manner, by means of periodic boundary conditions either in the frequency or the time-domain [6], [7]. Yet, the interaction of such structures as substrate layers with non-periodic metallic and dielectric elements of the geometry they are integrated with, as well as localized voltage and current sources, gives rise to typically large computational problems. A simple example along these lines is provided by a microstrip line printed on an electromagnetic band-gap substrate. This problem was efficiently addressed in [8], through an integral equation technique that combined a periodic Green’s function for the substrate with the array-scanning method [9]. In this formulation, the surface current density on the microstrip is treated as a non-periodic source. The use of a periodic Green’s function transforms this source into an array of sources, while the array-scanning method formally isolates the effect of the original microstrip current.

In the context of the the Finite-Difference Time-Domain (FDTD) method, the interaction of a non-periodic source with a periodic substrate has been recently considered [10]–[12], embedding the array-scanning method into periodic FDTD techniques (either the spectral FDTD of [13] or the sine-cosine method [6]). However, for applications involving microstrips or coplanar waveguides on periodic substrates and multi-layers, a crucial difference between FDTD and the integral equation method of [8] has to be pointed out. In FDTD, surface current densities on metallic guides are not modeled as primary sources, while field components tangential to metallic surfaces are set to zero. Hence, if the computational domain is terminated in periodic boundary conditions, these metallic surfaces are periodically reproduced; their presence cannot be eliminated by array-scanning. The aforementioned situation is demonstrated in Fig. 1, where \( \phi_0 \) represents the phase progression for a Floquet mode across one unit cell.

![Fig. 1. The problem of simulating a microstrip over a periodic substrate in FDTD with periodic boundary conditions: array-scanning eliminates the effect of the periodic sources, but not that of the strip boundary conditions (contrary to the integral equation technique [8]).](image)

This highly practical class of problems is addressed in this paper, within the framework of periodic FDTD formulations. First, it is shown that the sine-cosine method [6] can be employed for the wide-band characterization of periodic structures, contrary to some perception that it can only be employed for monochromatic simulations [14]. Then, the configuration of Fig. 2 is proposed as an alternative for the simulation of metallic integrated waveguide structures on periodic substrates. While previous research on periodic FDTD formulations has focused on the application of either periodic or absorbing boundary conditions at each boundary of a given computational domain, the problem at hand is best served by terminating the substrate...
at periodic boundary conditions and the space above it, including the nodes of the metallic guide, in absorbing boundary conditions (or perfectly matched layers). It is shown that this combination can be performed in a stable and efficient manner. Thus, the periodic imaging of the metallic boundaries is prevented, while the array scanning method can still be employed to isolate the effect of the original source excitation. In the following, the proposed methodology is briefly explained and applied to realistic electromagnetic band-gap based planar structures. The numerical results of the technique are evaluated against measurements, while its performance is compared to commercial simulators, to clearly demonstrate its feasibility and usefulness.

II. METHODOLOGY

A. The sine-cosine method: A new perspective

Consider a field expansion in terms of Floquet modes in a periodic structure, inverse Fourier transformed from the frequency to the time-domain:

\[
E(\tau, t) = Re \sum_p e^{-j\bar{k}_p \cdot \tau} \frac{1}{2\pi} \int \omega(\bar{k}_p) e^{-j\omega t} d\omega
\]

\[
= Re \sum_p e^{-j\bar{k}_p \cdot \tau} E(\bar{k}_p, t)
\]

\[
= Re \sum_p \left\{ E^p_C(\tau, t) - j E^p_s(\tau, t) \right\}
\]

where \(\omega(\bar{k}_p)\) is an either discrete or continuous spectrum of frequencies corresponding to the Floquet wavevector \(\bar{k}_p\) and:

\[
E^p_C(\tau, t) = \cos (\bar{k}_p \cdot \tau) E(\bar{k}_p, t)
\]

\[
E^p_s(\tau, t) = \sin (\bar{k}_p \cdot \tau) E(\bar{k}_p, t)
\]

Note that these two waves have identical frequency spectra (as they share a common temporal dependence), while elementary trigonometric identities can be used to show that:

\[
E^p_C(\tau + \bar{d}, t) = \cos (\bar{k}_p \cdot \bar{d}) E^p_C(\tau, t) - \sin (\bar{k}_p \cdot \bar{d}) E^p_s(\tau, t)
\]

\[
E^p_s(\tau + \bar{d}, t) = \sin (\bar{k}_p \cdot \bar{d}) E^p_C(\tau, t) + \cos (\bar{k}_p \cdot \bar{d}) E^p_s(\tau, t)
\]

where \(\bar{d}\) is the lattice vector of the periodic structure. Therefore, the Floquet waves \(E^p_C(\tau, t), E^p_s(\tau, t)\) are shown to satisfy the “sine-cosine” boundary conditions of [6]. This formulation offers new insights into the sine-cosine method. Clearly, these two waves are neither monochromatic nor at phase quadrature in time (in fact, our sine/cosine waves are distinguished based on their spatial dependence), as stated in original interpretations of the method [14]. Therefore, they can be excited by identical Gaussian sources, provided that the frequency spectrum of such sources includes \(\omega(\bar{k}_p)\). With \(E^p_C(\tau, t), E^p_s(\tau, t)\) being excited (in their respective meshes), their spectral analysis yields all frequencies \(\omega(\bar{k}_p)\) at once. These conclusions have been confirmed through extensive numerical experiments.

Outside the periodic substrate, the domain is terminated in perfectly matched layers (Fig. 2). Hence, the two sine/cosine meshes have to be set-up only for the substrate, thus reducing the performance cost of the complex formulation involved with this method of applying periodic boundary conditions.

B. Sine-cosine based array-scanning

Consider for simplicity a one-dimensional periodic substrate with lattice vector \(\bar{d} = d_x \hat{x}\). Let \(\mathbf{E}_{array}(\tau_0, k_x, t)\) be the electric field determined by the sine-cosine method, for a point \(\tau_0\) within the computational domain under study (consisting of one unit cell of the periodic substrate in the direction of periodicity), where \(-\pi/d_x \leq k_x \leq \pi/d_x\). The electric field \(\mathbf{E}_0\) at this point that is only due to the original source can be found by integrating over \(k_x\) [9]:

\[
\mathbf{E}_0(\tau_0, t) = \frac{d_x}{2\pi} \int_{-\pi/d_x}^{\pi/d_x} \mathbf{E}_{array}(\tau_0, k_x, t) dk_x
\]

Since (4) is a continuous integral, while only \(N\) discrete \(k_x\) points are sampled, (4) is approximated at a time \(t = m\Delta t\) (the \(m\)-th time-step of FDTD) by the sum:

\[
\mathbf{E}_0(\tau_0, m\Delta t) \approx \frac{1}{N} \sum_{n=-N/2}^{N/2} \mathbf{E}_{array}(\tau_0, \frac{2\pi n}{Nd_x} m\Delta t)
\]

Typically, the convergence of (5) is attained by 16-32 \(k_x\) points within the Brillouin zone. Further reduction in the number of points needed can be achieved through field interpolation with respect to \(k_x\), to enhance the spectral sampling rate in the Brillouin zone. However, it should be noted that the computation of each \(\mathbf{E}_{array}(\tau_0, k_x, t)\) can be performed independently. Therefore, the computation of \(\mathbf{E}_{array}(\tau_0, k_x, t)\) for multiple \(k_x\)'s is readily parallelizable.

III. APPLICATIONS

A. Slow-wave microstrip line over a periodic ground plane

The first example is a microstrip line with a periodically patterned ground plane, intended to produce a slow-wave...
guide [4]. The unit cell of the ground plane is depicted in Fig. 3(a), where the shaded area represents a perfect electric conductor. The unit cell size is 3.05 mm by 3.05 mm. The ground plane resides beneath a substrate with thickness of 0.635 mm and relative permittivity of \( \epsilon_r = 10.2 \). A microstrip line, 0.635 mm wide, is printed on the substrate as shown in Fig. 3(b).

In the FDTD simulation, a single unit cell is modeled by 48 \( \times \) 48 \( \times \) 22 cells, with the cell size of \( 0.064 \times 0.064 \times 0.212 \) mm. A layer of free space with 6 FDTD cells is added beneath the ground plane pattern to incorporate the interaction between the ground plane slits and air. For the analysis of the structure through the proposed technique, only one unit cell of the substrate in the \( x \)-direction is simulated. In the \( y \)-direction, 19 cells are used. The domain is excited by a \( z \)-polarized voltage source with a 0.5 to 10 GHz Gabor pulse, located underneath the microstrip line at the boundary of the 1st and 2nd cell in the \( y \)-direction. The computational domain is terminated in periodic boundary conditions in the \( x \)-direction within the area of the substrate (from the 7-th to the 9-th FDTD cell in the \( z \)-direction) and by a ten cell uniaxial perfectly matched layer elsewhere. Finally, the time step was set to 0.121 ps and 32,768 steps were run.

For the array-scanning method, 32 \( k_x \) points within the Brillouin zone (uniformly distributed every 0.0625\( \pi \) rad/m) were calculated in parallel, using a cluster environment. Scattering parameter results are plotted in Fig. 4. For comparison purposes, a finite version of the structure is also simulated with Ansoft’s HFSS (using seven cells in the \( x \)-direction). Moreover, measured results from [4] are appended. The agreement between HFSS and this work is excellent for both \( S_{11} \) and \( S_{21} \), although the proposed approach has a reduced simulation time by a factor of 3.8 (19.1 hrs vs. 72 hrs). Both sets of data are also in satisfactory agreement with the measured results of [4], taken over a structure that is truncated to seven cells in the \( x \)-direction.

### B. Microstrip over an electromagnetic band-gap structure

The second numerical example is a microstrip line on a substrate that is terminated at a ground plane with periodically etched circular apertures, which behaves as an electromagnetic band-gap [3]. Figure 5(a) shows the top view of one unit cell with the microstrip line. The size of the unit cell is 5.08 mm by 5.08 mm. The substrate of the structure is 0.635 mm thick with a relative permittivity of \( \epsilon_r = 10.5 \), and the width of the microstrip line is 0.686 mm. Figure 5(b) shows a three-dimensional view of the simulated structure.

The sine-cosine FDTD model of the unit cell includes 30 \( \times \) 30 \( \times \) 22 cells, with the cell size of 0.171 mm \( \times \) 0.171 mm \( \times \) 0.212 mm. Again, a layer of free space with 6 cells is added beneath the perforated ground plane. Only one unit cell of the substrate in the \( x \)-direction is used in FDTD, with 9 unit cells in the \( y \)-direction of signal propagation. Similar to the first example, the domain is excited by a \( z \)-polarized voltage source of a 4-14 GHz Gabor pulse, located underneath the microstrip line and at the boundary of the 1st and 2nd cell in the \( y \)-direction. The computational domain within the substrate is terminated in periodic boundary conditions in the \( x \)-direction and within the area of the substrate and by a 10 cell uniaxial perfectly matched layer elsewhere. The time step was set to 0.121 ps, and 16384 time steps were run.

Figure 6 shows the scattering parameters for the structure calculated by the proposed technique as well as a finite structure simulation with Ansoft’s HFSS, using three unit cells in the transverse direction. The FDTD analysis was completed in 3855 sec, while the finite structure simulation of HFSS took much longer (15 hrs), although just three
The use of the proposed approach circumvents the need for simulations of truncated periodic structures, using a finite number of unit cells, which are typically expensive and whose convergence is strongly problem-dependent. Hence, this work offers a useful analysis and design tool for substrate engineering studies, at a time when applications of novel ideas on microwave (meta-)materials are being intensively investigated.

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REFERENCES


IV. CONCLUSIONS

A technique to efficiently analyze microstrip-based microwave integrated circuit geometries on periodic substrates in the time-domain has been presented, extending the applicability of periodic FDTD formulations to this highly practical class of guiding and radiating structures.