Abstract—The Finite-Difference Time-Domain (FDTD) method is used to simulate the electromagnetic problem of transmission through a metallic screen with slots of sizes and spacing that were chosen using the spatially shifted beam approach to achieve subwavelength focusing. The challenges related with the modeling of such sub-wavelength focusing meta-screens are demonstrated via the study of numerical convergence.

I. INTRODUCTION

Subwavelength focusing of electromagnetic waves has gained renewed interest in recent years, largely due to a 2000 paper by J. B. Pendry suggesting the concept of a “perfect lens” [1]. In the “perfect lens”, as in all other methods for achieving subwavelength focusing, the goal is to overcome the diffraction limit, which is imposed by the decay of evanescent field components carrying high spatial resolution information. The “perfect lens” is made out of an artificial material that amplifies the evanescent field components as to compensate for their decay outside the lens. This idea was successfully demonstrated experimentally [2]– [4], but the artificial materials suffered from great losses. Another route to achieve subwavelength focusing is by synthesis of the field passing through a transmission mechanism with subwavelength features [5], [6]. This concept was taken a substantial step forward recently, when a simple and intuitive approach for the synthesis of source configurations that yield subwavelength focusing was presented in [7]. The method is named “spatially shifted beam approach” and is based on forming an expansion of target field distributions using near-field beam patterns created by displaced sources. The method was demonstrated in [7] using slots in a metallic screen as secondary sources, and the subwavelength focusing capability was measured by the beamwidth of the field pattern observed on a plane displaced 0.15λ (λ being the wavelength of operation) from the slotted screen.

As was observed in [7], the beamwidth of the observed field pattern is sensitive to the frequency of operation. Such a situation necessitates a large number of numerical results at different frequencies to better understand the behavior of the structure and offer additional design guidelines. The Finite-Difference Time-Domain (FDTD) method is a good candidate for such electromagnetic problems, both because a multitude of frequency domain results can be obtained from one simulation in the time-domain, and because, within the time-domain framework, the FDTD method has established itself as an effective and versatile solver of Maxwell’s equations. In this paper, we describe an FDTD based solution of the electromagnetic problem based on the structure studied in [7]. This work comes as an initial step towards the goal of shedding light on factors affecting the accuracy and convergence of the FDTD method when simulating subwavelength focusing structures in general and meta-screens in particular.

The remainder of this paper is organized as follows. The problem of transmission through a slitted screen is specified in Section II. In Section III, a formulation of the FDTD method for the above problem is given. In Section IV, results obtained with different meshes are presented and discussed. Finally, a summary and conclusions are given in Section V.

II. PROBLEM SPECIFICATION

We consider the problem of transmission of a plane wave through a highly conductive metallic screen with two possible configurations of slots. In the first configuration (henceforth, configuration A), there is only one slot of height \( h_{cen} = 13.2 \text{ mm} \) and width \( w_{cen} = 1.2 \text{ mm} \). In the second configuration (henceforth, configuration B), two satellite slots are added in a parallel-broadside configuration (see Fig. 1) on both sides of the slot of configuration A. The satellite slots are of a height of \( h_{sat} = 17 \text{ mm} \) and a width of \( w_{sat} = 0.6 \text{ mm} \), and their centers are displaced from the center of the central slot by \( d = 3 \text{ mm} \). The medium at both sides of the screen is free space. The plane wave is impinging on the metallic screen at normal incidence and the electric field is oriented perpendicular to the broad side of the slots (see Fig. 1). The choice of slot configuration and dimensions follows that in [7]. Those parameters were obtained via iterative optimization aimed at minimizing the beamwidth of the transmitted near-field pattern for operation at 10 GHz to achieve subwavelength focusing.

III. FDTD FORMULATION

The conventional FDTD method with Yee’s grid is used to simulate the medium on both sides of the metallic screen. The metallic screen is approximated by a perfect electric conductor (PEC) on the \( z = 0 \) plane. The excitation is implemented as a transparent plane-wave source located at a negative \( z \) position and is temporally shaped as a short Gaussian pulse.
(with a full width at half maximum (FWHM) of 5.3 GHz) that is modulated around 10 GHz. The simulation volume is terminated in the ±z directions with uniaxial perfectly matched layers (UPMLs). To circumvent the inclusion of the effects of a finite-size screen or plane-wave source, while minimizing the simulated volume, the terminations in the x and y directions on the two sides of the metallic screen were chosen to be different. On the z < 0 side of the screen, the simulation volume is terminated in the ±x directions by PECs and in the ±y directions by perfect magnetic conductors (PMCs). This choice of terminations allows the simulation of an infinite plane-wave incidence, while keeping the z < 0 side of the simulation volume small. In the z > 0 side of the screen, the simulation volume is terminated in the x and y directions by UPMLs to avoid reflections. To improve the efficiency of the FDTD solver, a logarithmic mesh grading scheme is used [8] (see Fig. 2). For ease of implementation, the cell size function obtained from the grading is separable \( \Delta_i(x, y, z) = \Delta_{i,x}(x)\Delta_{i,y}(y)\Delta_{i,z}(z) \), with \( i \in \{x, y, z\} \). The mesh grading algorithm divides the screen into segments based on the slot configuration. Within these segments it automatically creates a graded mesh starting with a predefined minimum cell size near the edges of the slots. The algorithm then logarithmically increases the cell size until reaching a maximum cell size on the other side of the segment. The rate of logarithmic increase is determined by a predefined maximum cell size that may be somewhat bigger than the actual maximum cell size due to the constraint of having an integer number of cells. For brevity, only the predefined maximum and minimum cell sizes will be given in the following discussion.

**IV. RESULTS**

For all the following simulations, the simulation volume is set as \( x \in [-8 \text{ mm}, 8 \text{ mm}] \), \( y \in [-17 \text{ mm}, 17 \text{ mm}] \), and \( z \in [-9 \text{ mm}, 7 \text{ mm}] \). The predefined maximum cell size in the x direction on the segments reaching the \( x = \pm 8 \) facets of the simulation volume is 0.49 mm. The predefined maximum cell size in the y direction on the segments reaching the \( y = \pm 17 \) facets of the simulation volume is 0.84 mm. The predefined maximum cell size in the z direction on the segment reaching the \( z = -9 \) facet of the simulation volume is 0.52 mm and on the segment reaching the \( z = 7 \) facet is 0.42 mm. The simulation volume and predefined maximum cell sizes on its facets were tuned by numerical experimentation to reduce the computational burden while keeping errors due to the grading and reflections from the UPMLs minimal. The predefined maximum cell size for all other segments in the simulations (inner segments that do not touch the facets of the simulation volume) was chosen as 0.1 mm. This value affects the maximum size of the cells between slot edges, and it was found that further decreasing this value does not change the simulation results. The size of the cells neighboring the slot edges is dictated by the values of the predefined minimum cell size in the \( x, y \) and \( z \) directions, which are denoted by \( \Delta_{\text{min},i} \), with \( i \in \{x, y, z\} \). To ensure stability, the time step \( \Delta t \) for all simulations is chosen by employing the Courant-Friedrichs-Lewy (CFL) number \( s = c\Delta t \sqrt{\Delta_{\text{min},x}^{-2} + \Delta_{\text{min},y}^{-2} + \Delta_{\text{min},z}^{-2}} = 0.95 \).

Figure 3 shows simulated and measured results for the normalized magnitude of \( E_x \) at 10 GHz for configuration A (one slot). Panel (a) shows results obtained (after normalizing to maximum value) on a measurement plane located 4.5 mm (0.15\( \lambda \)) from the metallic screen. The simulated results are obtained through a Fourier transform of data from a 51,970 time steps long (a simulated time span of roughly 5.5 ns) FDTD simulation. The predefined minimum cell dimensions used for the simulation are \( \Delta_{\text{min},x} = \Delta_{\text{min},z} = 0.05 \text{ mm} \) and \( \Delta_{\text{min},y} = 0.1 \text{ mm} \), yielding a volume comprising 116 \( \times 212 \times 105 \) Yee cells. The measured results in panel (a) are taken from [7], and can be seen to be reasonably close to the FDTD simulation results. Panel (b) of Fig. 3 shows the
The \( E_x \) field magnitudes on the \( y = 0 \) plane. To highlight the profile of the beam, the field magnitudes are normalized along the \( x \) direction with respect to the maximum field value at \( x = 0 \). The contour in panel (b) of Fig. 3 depicts the FWHM beamwidth of the \( E_x \) field as a function of the \( z \) coordinate. The dotted horizontal line in panel (b) indicates the position of the measurement plane on which the results of panel (a) are obtained.

In a manner similar to the presentation of the results for configuration A in Fig. 3, Figure 4 presents simulated and measured results for the normalized magnitude of \( E_x \) at 10 GHz for configuration B (three slots). Panel (a) in this figure also shows results obtained (after normalizing to maximum value) on a measurement plane located 4.5 mm (0.15\( \lambda \)) from the metallic screen. For the case of the three slots, the mesh resolution has a substantial effect on the results obtained, and thus, different plots are shown for different meshes. The parameters dictating the mesh resolution are the predefined minimum cell dimensions \( \Delta_{\text{min},i} \), \( i \in \{x, y, z\} \). Since the incident electric field is directed in \( x \) direction, edge effects are expected to be more pronounced in this direction, and in turn, require higher mesh resolution. Indeed, numerical experimentation showed that a resolution given by \( \Delta_{\text{min},y} = 0.1 \text{ mm} \) is sufficient for accurate representation of the fields, while higher resolutions in the \( x \) direction (lower values of \( \Delta_{\text{min},x} \)) are needed. We, therefore, fix \( \Delta_{\text{min},y} = 0.1 \text{ mm} \) and show in Fig. 4 different plots for results obtained with different values of \( \Delta_{\text{min},x} = \Delta_{\text{min},z} \). Reducing \( \Delta_{\text{min},z} \) together with \( \Delta_{\text{min},x} \) stems from the need to accurately represent the increased exponential decay in the \( z \) direction that corresponds to higher resolution in the \( x \) direction.

Results for all the plots Fig. 4 are obtained through a Fourier transform of data from an FDTD simulation with a time span of roughly 5.5 ns. The simulation parameters are summarized in Table I, where \( N_t \) stands for the number of simulated time steps and \( N_x \), \( N_y \), and \( N_z \) stand for the total number of Yee cells in the \( x \), \( y \), and \( z \) directions, respectively. The measured results in panel (a) of Fig. 4 are also taken from [7]. Note that as the mesh resolution increases, the beam shape narrows.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \Delta_{\text{min},x} ) (mm)</th>
<th>( \Delta_{\text{min},y} ) (mm)</th>
<th>( \Delta_{\text{min},z} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta t )</td>
<td>0.18 ps</td>
<td>0.11 ps</td>
<td>0.06 ps</td>
</tr>
<tr>
<td>( N_t )</td>
<td>30,000</td>
<td>51,970</td>
<td>99,500</td>
</tr>
<tr>
<td>( N_x \times N_y \times N_z )</td>
<td>124 \times 238 \times 86</td>
<td>168 \times 238 \times 105</td>
<td>206 \times 238 \times 124</td>
</tr>
</tbody>
</table>
Fig. 5. FWHM beamwidth on the measurement plane vs. frequency around 10 GHz with different predefined minimum cell dimensions in the x and z directions ($\Delta_{\min,x}, \Delta_{\min,z}$) for (a) Configuration A (one slot) and (b) configuration B (three slots). The inset in panel (b) is a zoom of the region marked by a thin dotted rectangle.

and becomes closer to that obtained in the measurements. As in Fig. 3, Panel (b) of Fig. 4 shows the normalized $E_z$ field magnitudes on the $y = 0$ plane. The three differently patterned contours in panel (b) of Fig. 3 depict the FWHM beamwidths of the $E_z$ field as a function of the $z$ coordinate for the same three different predefined minimum cell dimensions used in panel (a).

It is clear from Figs. 3 and 4 that the convergence of the simulated results with the increase of the mesh resolution is much slower in the case of configuration B when compared to configuration A. To better understand the reason for this difference, we plot the FWHM beamwidth on the measurement plane as a function of frequency in the vicinity of 10 GHz for both configurations in Fig. 5. Note that 10 GHz is the frequency for which the three slots in configuration B were optimized in [7] to achieve a narrow beam pattern. In both panels of Fig. 5, different plots correspond to simulations with different predefined minimum cell dimensions in the x and z directions ($\Delta_{\min,x}, \Delta_{\min,z}$). Panel (a) shows results for configuration A while panel (b) shows results for configuration B. The results for configuration A overlap and indicate that a predefined minimum cell dimension of $\Delta_{\min,x} = \Delta_{\min,z} = 0.1$ mm suffices for obtaining accurate results. For this mesh, the volume comprises $94 \times 212 \times 86$ Yee cells and the number of time steps used is 30,000 (as in all previous simulations, this results in a simulated time span of roughly 5.5 ns). Increasing the number of time steps for this configuration does not change the results. From panel (b) it is evident that configuration B exhibits a resonant-like behavior in the beamwidth that is trimmed from above due to the finite volume of simulation. The fact that this phenomenon does not exist in configuration A and that it reduces the beamwidth in the vicinity of the intended frequency of operation, 10 GHz, indicates that it was created by the optimization of the three slots for subwavelength focusing. From the magnification of the region around 10 GHz, it is clearly seen that the position of the resonance-like shape is modified by a change in the mesh resolution. Intuitively, the sharpness of the resonance-like shape is the cause for the high sensitivity to mesh resolution. Studying the convergence of the beamwidth when increasing the number of simulated time steps, we found that close to the center of the resonance-like shape the convergence becomes slow. This implies that for higher mesh resolutions, where the resonance-like shape is shifted towards the frequency of operation, more time steps may be needed in order to obtain accurate results.

V. CONCLUSION

In this paper the FDTD method has been applied to the simulation of transmission through a metallic screen with slots of sizes and spacing that were optimized using the spatially shifted beam approach to achieve subwavelength focusing. A simple mesh grading scheme was applied to allow the study of the convergence of the results as the mesh resolution in the vicinity of the slot edges is increased. It was found that the optimization for subwavelength focusing introduces a sharp resonance-like behavior close to the desired frequency of operation that hinders the convergence with mesh resolution. It was also found that, in resemblance to the simulation of resonance structures with high quality factors where the near-fields are built slowly in time, results obtained in a frequency close to the center of the sharp resonance-like shape require a large number of simulation time steps to converge. Future study will be aimed at obtaining efficient FDTD based schemes with faster convergence rates when examining frequencies close to the center of the resonance-like shape of subwavelength focusing devices such the meta-screens described above. Another aim is to alter these FDTD schemes to allow efficient study of such devices at frequencies where metals exhibit plasmonic behavior.

REFERENCES