Efficient Finite-Difference Time-Domain Modeling of Driven Periodic Structures and Related Microwave Circuit Applications

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Abstract—The sine–cosine method for the finite-difference time-domain-based dispersion analysis of periodic structures is extended to incorporate the presence of nonperiodic wideband sources. A new formulation of this method is presented to clearly demonstrate that it can be employed for the characterization of periodic structures over a broad bandwidth. Moreover, its coupling with the array-scanning technique enables the incorporation of nonperiodic sources, thus enabling the fast characterization of driven periodic structures in the time domain via a small number of low-cost simulations. The convergence, accuracy, and efficiency of the proposed method is demonstrated with its application to the analysis of a negative-refractive-index transmission-line “perfect lens” and the successful comparison of simulated with experimental results. Finally, a modified version of this method is proposed for the accelerated simulation of microwave circuit geometries printed on periodic substrates.

Index Terms—Finite-difference time-domain (FDTD) methods, microstrip circuits, periodic structures.

I. INTRODUCTION

T HE STUDY of periodic structures is motivated by the many applications they can support, either as frequency-selective surfaces [1], photonic- and electromagnetic-bandgap crystals [2], [3], or artificial dielectrics [4]. The interest in the latter area has been recently enhanced by the extensive research activity aimed at synthesizing media with unusual macroscopic properties (metamaterials [5]). Along with this activity, numerical tools that can capture unconventional wave effects observed in metamaterial geometries, such as negative refraction, and illuminate the underlying physics have been proposed. To this end, time-domain techniques, such as the finite-difference time-domain (FDTD) method [6], are particularly useful because they effectively model the rich transients involved with the evolution of these effects. The potential of the FDTD method to significantly contribute to understanding the nature of wave propagation in synthesized media has been demonstrated in several papers, including [7]–[10].

Furthermore, the dispersion analysis of periodic structures can be carried out by simulating a single unit cell of those, terminated with periodic boundary conditions. Originally cast in the frequency domain, periodic boundary conditions can be translated into the framework of the FDTD method [6]. In [11], the well-known sine–cosine method [12] was employed to analyze a recently proposed 2-D negative-refractive-index transmission-line structure [13]. In [14], this method was extended to account for leaky-wave radiation from the same structure, indicating an efficient FDTD-based methodology for the concurrent computation of attenuation and phase constants of fast waves in periodic geometries. Finally, in an effort to investigate the possibility of transferring the concepts of negative-refractive-index transmission lines from the microwave to the optical regime (along the lines of [15]), a conformal periodic FDTD analysis of plasmonic nanoparticle arrays in a triangular mesh was presented in [16].

Building on this earlier work and the research reported in [17], the problem of modeling driven periodic structures within the same simulation framework that employs the sine–cosine method to implement periodic boundary conditions is considered in this paper. Since the presence of a nonperiodic source is not compatible with the use of periodic boundary conditions, this problem would be typically handled by simulating a finite version of the periodic structure, up to the number of cells necessary to achieve the convergence of the solution. Evidently, the efficiency of this approach largely depends on the nature of the problem at hand and may be quite costly in terms of execution time and computer memory. A similar question, arising in the context of the method of moments, was addressed in [18] by invoking the array-scanning method of [19], to model the interaction of a printed microstrip line with an electromagnetic bandgap substrate. Recently, [17] suggested that the same methodology can enable the modeling of driven periodic structures by means of the sine–cosine method. Independently, Yang et al. [20] and Qiang et al. [21], [22] combined a spectral FDTD method with array scanning, thus presenting an alternative methodology for the same class of problems.

In particular, this paper makes the following contributions. First, it is rigorously shown that the sine–cosine method of [12] can be applied for the broadband characterization of periodic structures, although it had been originally suggested that its applicability was limited to monochromatic simulations [6]. On the contrary, a new formulation of the method offers new insights to its broadband character and the sources necessary to excite Floquet modes in a sine–cosine-based FDTD mesh. This
paves the way for the coupling of the sine–cosine with the array-scanning technique, which results in an efficient modeling tool for the interaction of broadband nonperiodic sources with periodic geometries based on a small number of low-cost simulations. The effectiveness of this tool is tested in the challenging problem of the negative-refractive-index transmission-line “perfect lens” [23], which is used as a vehicle for the demonstration of its accuracy and convergence properties. Finally, the application of the proposed technique to the modeling of microwave circuits printed on periodic substrates is discussed. It is shown that the presence of aperiodic metallic boundaries (in addition to the source that excites the microstrip) cannot be accounted for by the array-scanning method alone. Instead, a composite boundary is proposed, where both periodic and absorbing boundary conditions are applied. Thus, a highly efficient simulation approach for this class of problems is offered.

II. ARRAY-SCANNING SINE-COSINE METHOD: FORMULATION AND WIDEBAND VALIDITY

A. Problem Statement

The problem under consideration is shown in Fig. 1, where the interaction of a broadband nonperiodic source with an infinite 2-D periodic geometry is shown. Instead of approximating the infinite periodic structure by a truncated version of it, the proposed solution is based on the computational domain of Fig. 2(a), where periodic boundary conditions are applied to the electric field phasors (denoted by *) at the boundaries along the two directions of periodicity

$$\mathbf{\tilde{E}}(\mathbf{r} + \mathbf{p}) = \mathbf{\tilde{E}}(\mathbf{r}) \exp(-j\mathbf{k}_p \cdot \mathbf{p}) \quad (1)$$

where $\mathbf{p} = d_x \mathbf{a}_x + d_y \mathbf{a}_y$ is the lattice vector of the periodic structure and $\mathbf{k}_p = k_x \mathbf{a}_x + k_y \mathbf{a}_y$ is a Floquet wave vector. However, the computational domain of Fig. 2(a) leads to the solution of the problem shown in Fig. 2(b), where the response of the structure to an array, consisting of phase-shifted periodic replicas of the original source is determined. In Fig. 2(b), $\phi_x = \kappa_x d_x, \phi_y = \kappa_y d_y$.

The derivations and numerical results of this section are aimed at showing that the problem of Fig. 2(a) can be solved by means of the sine–cosine method [12], and to clarify the source conditions needed. Moreover, the sine–cosine method is coupled with the array-scanning technique to isolate the effect of the original source from the combined effect of the phased array of sources shown in Fig. 2(b).

B. New Derivation for the Sine–Cosine Method

Consider a field expansion in terms of Floquet modes in a periodic structure of lattice vector $\mathbf{p}$, inverse Fourier transformed from the frequency to the time domain

$$\mathbf{E}(\mathbf{r}, t) = \text{Re} \sum_{\mathbf{k}} e^{-j\mathbf{k}_p \cdot \mathbf{r}} \frac{1}{2\pi} \int_{\omega(\mathbf{k}_p)} \mathbf{E}(\mathbf{r}, \omega) e^{-j\omega t} d\omega$$

$$= \text{Re} \sum_{\mathbf{k}} e^{-j\mathbf{k}_p \cdot \mathbf{r}} \mathbf{E}(\mathbf{r}, t)$$

$$= \text{Re} \sum_{\mathbf{k}} \left\{ \mathbf{\tilde{E}}(\mathbf{r}, t) - j \mathbf{\tilde{E}}(\mathbf{r}, t) \right\} \quad (2)$$

Fig. 1. Geometry of the problem under consideration: a nonperiodic source exciting a 2-D infinite periodic structure of spatial period $d_x$ along the $x$-axis.
where $\omega(\vec{k}_p)$ is an either discrete or continuous spectrum of frequencies corresponding to the Floquet wave vector $\vec{k}_p$, and

$$
\begin{align*}
E_C^p(\vec{r}, t) &= \cos(\vec{k}_p \cdot \vec{r}) \vec{E}(\vec{k}_p, t) \\
E_s^p(\vec{r}, t) &= \sin(\vec{k}_p \cdot \vec{r}) \vec{E}(\vec{k}_p, t)
\end{align*}
$$

(3)

Note that these two waves have identical frequency spectra (as they share a common temporal dependence). Moreover,

$$
\begin{align*}
E_C^p(\vec{r} + \vec{p}, t) &= \cos(\vec{k}_p \cdot \vec{r} + \vec{k}_p \cdot \vec{p}) \vec{E}(\vec{k}_p, t) \\
E_s^p(\vec{r} + \vec{p}, t) &= \cos(\vec{k}_p \cdot \vec{r}) \cos(\vec{k}_p \cdot \vec{r}) \vec{E}(\vec{k}_p, t) \\
&- \sin(\vec{k}_p \cdot \vec{r}) \sin(\vec{k}_p \cdot \vec{r}) \vec{E}(\vec{k}_p, t) \\
&= \cos(\vec{k}_p \cdot \vec{r}) E_C^p(\vec{r}, t) - \sin(\vec{k}_p \cdot \vec{r}) E_s^p(\vec{r}, t)
\end{align*}
$$

(4)

Similarly,

$$
\begin{align*}
E_s^p(\vec{r} + \vec{p}, t) &= \sin(\vec{k}_p \cdot \vec{r}) E_C^p(\vec{r}, t) + \cos(\vec{k}_p \cdot \vec{r}) E_s^p(\vec{r}, t).
\end{align*}
$$

(5)

Therefore, the Floquet waves $E_C^p(\vec{r}, t), E_s^p(\vec{r}, t)$ are shown to satisfy the “sine–cosine” boundary conditions of [12]. Note that it is straightforward to implement (4) and (5) in the discrete-time framework of the FDTD since all terms on the two sides of the equation are evaluated at the same time step. This formulation offers new insights into the sine-cosine method. Clearly, these two waves are neither monochromatic, nor at phase quadrature in time. In fact, our sine/cosine waves are distinguished based on their spatial rather than temporal dependence. Therefore, they can be excited by identical broadband sources (instead of sine/cosine modulated ones), provided that the frequency spectrum of such sources includes $\omega(\vec{k}_p)$. With $E_C^p(\vec{r}, t), E_s^p(\vec{r}, t)$ being excited (in their respective meshes), their spectral analysis yields all frequencies $\omega(\vec{k}_p)$ at once. This is demonstrated through the numerical results of Section II-C.

C. Wideband Validity of the Sine–Cosine Method: Numerical Results

Consider the unit cell of the 2-D negative-refractive index transmission-line structure that was originally presented in [13], shown in Fig. 3(a). The corresponding positive-refractive index transmission-line unit cell is also appended in Fig. 3(b). This unit cell resides on a substrate of thickness 1.52 mm and relative permittivity $\varepsilon_r = 3$. The spatial periods $d_x$ and $d_y$ (indicated in Fig. 3) are both equal to 8.4 mm. The width $w$ of the microstrip lines is 0.75 mm. In the FDTD mesh, the negative-refractive index unit cell is discretized by $22 \times 22 \times 16$ Yee cells. Three of the 16 cells in the z-direction model the substrate. The open boundary in the vertical direction is simulated by a uniaxial perfectly matched layer (PML) absorber [6]. This absorber consists of ten cells with a fourth-order polynomial conductivity grading. The maximum conductivity value is $\sigma_{\text{max}} = 0.01194/\Delta$ with $\Delta$ being the Yee cell size in the direction of mesh truncation (hence, in this case, the open boundary being parallel to the $x$–$y$ plane, $\Delta = \Delta_z$). The same absorber has been used to simulate open boundaries in all numerical simulations included in this paper.

Moreover, the series capacitor and shunt inductor, shown in Fig. 3(a), are chosen to be $C = 3.34 \, \text{pF}$ and $L = 16.12 \, \text{nH}$. The two sine–cosine grids are excited by a 0.5–3-GHz Gabor pulse

$$
\exp\left(\frac{t - t_0}{\tau_w}\right)^2 \sin(2\pi f_c t)
$$

applied to the $E_z$ components in cells $(6,11,1),(6,11,2)$, and $(6,11,3)$ inside the substrate. The Gabor pulse parameters are $\tau_w = 624 \, \text{ps}$ and $t_0 = 3\mu t$. The time step is set to 0.723 ps and 60 000 time steps are performed for three cases of $k_{y,\text{max}}d_y = 0.0833\pi, 0.167\pi, 0.333\pi$, while $k_{y,0} = 0$. Hence, all three points are along the $\Gamma$–$X$ portion of the Brillouin diagram of the structure that is occupied by three TM waves, as shown in previous studies as well [11]: a backward, forward, and surface wave. In Fig. 4(a)–(c), the $\Gamma$–$X$ part of the Brillouin diagram for the negative-refractive index transmission-line unit cell, independently determined by Ansoft’s High Frequency Structure Simulator (HFSS), is shown along with the magnitude of the Fourier transform (normalized to its maximum) of a vertical electric field component $E_z$ determined by the sine–cosine FDTD method and sampled within the substrate from 0 to 5 GHz. For each case of $k_{y,\text{max}}d_y$, the FDTD-calculated field presents multiple resonances, which correspond to the frequencies $\omega(k_{y,\text{max}}d_y)$, given by the intersections of the diagram with the constant $k_{y,\text{max}}d_y$ lines. Hence, the FDTD and HFSS calculated resonant frequencies are in excellent agreement.

Moreover, it is clearly shown that a single run of the sine–cosine FDTD, with the same excitation for each grid, is sufficient to determine all resonant frequencies at once. Note that the boundary conditions (4) and (5) enforce the Floquet wave
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D. Coupling the Array-Scanning and Sine-Cosine Methods

The combination of periodic boundary conditions with a broadband source leads to the solution of the problem shown in Fig. 2(b), where the response of the structure to an array, consisting of phase-shifted periodic repetitions of the original source, is determined. It is the purpose of the array-scanning technique to isolate the effect of the original source, as described below.

Let \( \mathbf{E}_{\text{array}}(\vec{r}_0, \vec{k}_p, t) \) be the electric field determined by the sine–cosine method, at a point \( \vec{r}_0 \) within the unit cell, for a Floquet wave vector \( \vec{k}_p = \delta \vec{k}_c + \vec{y} \vec{k}_y \) within the Brillouin zone of the structure (hence, \(-\pi/d_x \leq k_x \leq \pi/d_x \) and \(-\pi/d_y \leq k_y \leq \pi/d_y \)). The electric field \( \mathbf{E}_0(\vec{r}_0, t) \) at this point that is only due to the original source can be found by integrating over \( k_x, k_y \) [19]

\[
\mathbf{E}_0(\vec{r}_0, t) = \frac{d_x d_y}{4\pi^2} \int_{-\pi/d_x}^{\pi/d_x} \int_{-\pi/d_y}^{\pi/d_y} \mathbf{E}_{\text{array}}(\vec{r}_0 + \vec{k}_p, t) dk_x dk_y.
\]

(6)

Since (6) is a continuous integral, while only \( N \) discrete \( k_x \) and \( M \) discrete \( k_y \) points are sampled, (6) is approximated at a time \( t = l\Delta t \) (the \( l \)th time step of the FDTD method) by the sum

\[
\mathbf{E}_0(\vec{r}_0, l\Delta t) \approx \frac{1}{NM} \sum_{n=-N/2}^{N/2} \sum_{m=-M/2}^{M/2} \mathbf{E}_{\text{array}}(\vec{r}_0 + \vec{k}_{ij}, l\Delta t),
\]

(7)

A modified form of (7) can be employed to determine the electric field at points outside the simulated unit cell by invoking the periodic boundary conditions (1). In particular,

\[
\mathbf{E}_0(\vec{r}_0 + \vec{p}_{kj}, l\Delta t)
\approx \frac{1}{NM} \sum_{n=-N/2}^{N/2} \sum_{m=-M/2}^{M/2} \mathbf{E}_{\text{array}}(\vec{r}_0 + \vec{k}_{ij}, l\Delta t) \exp(-j\vec{k}_{p} \cdot \vec{p}_{kj})
\]

(8)

with \( \vec{p}_{kj} = \vec{x} \delta d_x + \vec{y} \delta d_y \) for integer \( i, j \).

The fundamental limit that guides the choice of the number of points \((N \times M)\), which need to be sampled inside the Brillouin zone, is the sampling theorem. If the fields in the driven periodic structure under study are spatially limited within the area \((-W_x \leq x \leq W_x, -W_y \leq y \leq W_y)\), then the sampling rates \( S_x = N/(2\pi/d_x) \) samples/(rad/m) and \( S_y = M/(2\pi/d_y) \) samples/(rad/m) should obey the inequalities

\[
2\pi S_x \geq 2W_x \quad 2\pi S_y \geq 2W_y
\]

(9)

which leads to

\[
N \geq 2\frac{W_x}{d_x}, M \geq 2\frac{W_y}{d_y}.
\]

(10)
Practically, safe bounds for $W_x$ and $W_y$ can be deduced from the physics of the problem at hand. For example, in the presence of a driven microstrip line printed over a periodic structure, the spatial extent of the fields can be estimated by the well-known Wheeler formula for the microstrip width correction due to field fringing. On the other hand, if there is significant field coupling in the direction of periodicity, a larger number of $N, M$ may be needed. A convergence study is then necessary. It is noted that all sine–cosine FDTD simulations for different $k_x$’s are independent from each other, and therefore, lend themselves to perfect parallelization with no inter-processor communication overhead. In consequence, in a parallel environment, there is no added cost as the number of sampled Floquet wave vectors increases. In the following examples, driven periodic structures of the artificial dielectric type (with $d_x, d_y \ll \lambda$) and of the electromagnetic-bandgap type (with $d_x, d_y \sim \lambda$) are examined. Since these are driven along one of the two directions of periodicity, they are treated as 1-D structures, where 10–20 $k_x$ points are always sufficient for convergence of the calculated fields.

III. ANALYSIS OF A NEGATIVE-REFRACTIVE-INDEX TRANSMISSION-LINE-BASED MICROWAVE “PERFECT LENS”

In this section, the proposed method is applied to a microwave implementation of Pendry’s concept of a “perfect lens” [25] that has been recently proposed and experimentally demonstrated [23]. The structure utilizes the unit cells of 2-D positive and negative-refractive-index transmission lines, shown in Fig. 3. The parameters of both cells and of their FDTD discretization are the same as in Section II.

To evaluate the convergence properties of the methodology of Section II, a domain consisting of 18 positive-refractive index transmission-line cells in the $y$-direction is used. The geometry is treated as a 1-D periodic structure in the $x$-direction and simulated by the sine–cosine method, using one unit cell along the direction of periodicity, terminated at periodic boundary conditions. The vertical electric field $(E_z)$ nodes within the substrate of the first cell are excited by a sinusoidal hard source at 1 GHz. The vertical electric field $E_z$ at the center of each subsequent cell is then determined in the middle of the substrate. The standard approach to this problem, namely, the use of a finite number of cells along the $x$-direction until a convergent solution is attained, has also been implemented. In both simulations, the time step is again set to 0.723 ps and the total number of time steps is 16384.

The results of the two approaches are presented in Figs. 5 and 6, respectively, which include diagrams of the computational domains used. It is noted that the sine–cosine method based array-scanning converges with $N = 16$ $k_x$ points, or a sampling rate of $0.125\pi$ rad/m in the wavenumber domain. On the other hand, 17 cells are needed for the field in the finite structure in the $x$-direction to converge within 1% of the fields of the infinite periodic one. The convergence properties of the two methods are summarized in Fig. 7, which depicts the relative error norm

$$\mathcal{E} = \sum_j \left( \frac{E_z(j) - E_z^{\text{ref}}(j)}{E_z^{\text{ref}}(j)} \right)^2$$  \hspace{1cm} (11)

where $E_z(j)$ is the $z$-component of the electric field in the middle of the substrate and along the $y$-axis, calculated with the array-scanning sine–cosine FDTD and finite structure simulations (plotted in Figs. 5 and 6, respectively) and $E_z^{\text{ref}}$ is the same field calculated with a 32-cell finite structure FDTD simulation. The error norm $\mathcal{E}$ is plotted with respect to the number of $k_x$ points used for the array-scanning-based field calculation and with respect to the number of cells in the transverse direction used for the finite structure field calculation.

The electric field amplitude decays away from the source, as expected. As discussed in [23], this amplitude decay, and the resulting loss of the evanescent spectral components of the source, can be compensated for by introducing a layer occupied by the negative-refractive-index transmission-line cells of Fig. 3(a). To achieve the matching of this layer to the positive-refractive index transmission-line half-spaces (to the left and right of it) at 1 GHz, the loading elements of Fig. 3(a) are
Fig. 7. Error norm $\mathcal{E}$ of (11) with respect to the number of $k_x$ points used for the array-scanning-based field calculation and with respect to the number of cells in the transverse direction used for the finite structure field calculation.

Fig. 8. Vertical electric field $E_z$ in the middle of the substrate and along the $x$-axis in a planar microwave lens geometry, calculated via the sine–cosine–based array-scanning method ($N = 16$) and a finite structure simulation, using 17 cells in the $x$-direction. All fields have been normalized to their maximum amplitude.

Fig. 9. Vertical electric field $E_z$ in the middle of the substrate and along the $x$-axis in a planar microwave lens geometry, calculated via the sine–cosine–based array-scanning method ($N = 16$) and a finite structure simulation, using 17 cells in the $x$-direction, at the source and the image (focal) plane. All fields have been normalized to their maximum amplitude.

$C = 3.34 \text{ pF}$ and $L = 16.0 \text{ nH}$, as in the case studies of Section II. The characteristic impedance of both lines then becomes $50 \Omega$. The domain is excited by a 1-GHz sinusoidal hard source that is placed 2 1/2 unit cells away from the first interface. Note that the negative-refractive-index region occupies five cells, twice as many as the distance of the source and the image plane from the positive-to-negative index interfaces.

The expected electric field ($E_z$) amplitude growth within the negative-refractive-index slab is verified by the sine–cosine-based array scanning, as shown in Fig. 8, which includes a diagram of the computational domain. For these results, 16 $k_x$ points have been calculated. For comparison, the results of a finite structure simulation, employing 17 cells in the transverse ($x$-direction) direction, are appended, being in good agreement with the sine–cosine-based array-scanning results. It is noted that the field amplitude growth effect is due to resonant coupling between the two interfaces, and therefore, builds up rather slowly during the time-domain simulation. The steady state is reached in 60,000 time steps, in a total simulation time of 2454 s with the sine–cosine-based array-scanning method, as opposed to 20513 s with the finite structure simulation. Hence, the value of reducing the computational domain of the problem is even higher in this case.

The field pattern of Fig. 8 indicates that the matching of the positive- and negative-refractive-index regions is imperfect, mainly because of the fringing capacitance at the microstrip gaps where the lumped capacitors are placed, which contributes to the total gap capacitance. As a result, the field growth starts outside the negative-refractive-index slab because of the interaction between incident and reflected waves, something also evident in the experimental results of [23]. The mismatch, along with the fact that the structure has a finite spatial period, leads to an imperfect restoration of the source at the image plane (2 1/2 unit cells from the second interface), which is still better than the conventional diffraction-limited case. Indeed, Fig. 9 shows the electric field ($E_z$) amplitude at the source and image plane (along the transverse direction), determined via the aforementioned sine–cosine-based array-scanning and finite structure simulations. The diffraction-limited source image (for an all positive-index space) is also appended. It is noted that the half-power beamwidth of the source image extends over four cells, whereas the diffraction-limited image extends over six cells. These patterns are in excellent agreement with the experimental results of [23] for the same structure and offer the first full-wave validation of those.
The vertical field values in the transverse direction, beyond the simulated unit cell, shown in Fig. 9 have been calculated by means of (8). Note that there are significant field values in up to approximately six unit cells in the ±x-direction. As a result, applying (10) with $W_x \approx 6d_x$ yields $N \geq 12$ as a limit for the number of $k_x$ points needed for the reconstruction of the field profile in the space domain, which is consistent with the results of our convergence study. This is an $a$ posteriori verification of the bounds of (10).

IV. MICROSTRIP LINE PRINTED ON AN ELECTROMAGNETIC-BANDGAP SUBSTRATE

A. Problem Statement

The second example is the electromagnetic-bandgap substrate microstrip line that was studied in [18]. The three-layer periodic substrate is shown in Fig. 10. All three layers are $h_1 = h_2 = h_3 = 0.635$ mm high and their dielectric constants are 9.8, 3.2, and 9.8, respectively. The center layer includes periodic rectangular air blocks of 6.5 mm × 6.5 mm × 0.635 mm. The spacing between the neighboring blocks is $\alpha = 14$ mm in both directions. The width of the microstrip line, which is aligned with the air blocks underneath it, is 3 mm.

This structure belongs to a highly practical class of problems, where nonperiodic planar waveguides are printed on periodic substrates. For such applications, a crucial difference between the FDTD and integral-equation method of [18] exists [26]. In the FDTD, surface current densities on metallic guides are not modeled as sources, while field components tangential to metallic surfaces are set to zero. Hence, if the computational domain is terminated in periodic boundary conditions, these metallic surfaces are periodically reproduced; their presence cannot be eliminated by array scanning. The aforementioned situation is demonstrated in Fig. 11, where $\phi_x$ represents the phase progression for a Floquet mode across one unit cell (in the $x$-direction).

To numerically confirm that this problem does accompany the application of the sine–cosine method, the following simulations are performed. First, a single unit cell of the periodic substrate is terminated with sine–cosine-based periodic boundary conditions in the $x$-direction, as shown in Fig. 11(b), and excited with a 2–10-GHz modulated Gaussian hard source. The $x$-component of the electric field is sampled at the plane of the air–substrate interface, which is the plane where the microstrip line lies as well. This is compared to the $E_x$ component in a structure consisting of seven unit cells of the periodic substrate in the $x$-direction, including the microstrip [similar to Fig. 11(d)]. The Euclidean norms of the two

$$||E_x||_2 = \sqrt{\sum_n |E_x(n\Delta t)|^2}$$

are shown to be identical in Fig. 12. The plot of this norm also clearly shows that the middle strip (and the associated boundary condition $E_x = 0$) is periodically reproduced by the periodic boundary conditions, after the application of array scanning (which, in [18], was sufficient to eliminate the presence of these strips). Hence, this important difference between integral-equation methods and the FDTD needs to be taken into account when the modeling of printed waveguides on periodic substrates is pursued through an approach that is presented in Section IV-B.

B. Proposed Methodology

While previous research on periodic FDTD formulations has focused on the application of either periodic or absorbing boundary conditions at each boundary of a given computational domain (a feature inherited by commercial packages as well), the problem at hand is best served by terminating the substrate at periodic boundary conditions and the space above it, including the nodes of the metallic guide, in absorbing boundary conditions (or PMLs). This approach corresponds to the configuration shown in Fig. 13. Thus, the periodic imaging of the metallic boundaries is prevented, while the array scanning.
A 2-D periodic geometry in the x-direction is mod-
emerged, terminated in sine–cosine periodic boundary condi-
tions within the substrate and the aforementioned PML,
the scattering parameters of five unit cells in the y-direction
are computed. One unit cell, without air blocks, is added
before and after these five cells, to provide space for the exci-
tion and probe points, giving rise to a domain of seven unit
38x491
structures in total, terminated at a PML as well. The Yee cell size
is 0.5 mm \times 0.5 mm \times 0.318 mm, and hence, a single unit cell
of the structure contains 28 unit cells in the trans-
sverse direction at the interface are not determined. Hence,
the one cell beneath the microstrip, as determined by the sine–co-
sine-based array-scanning analysis method can still be employed to isolate the effect of the original
source excitation.

When a PML absorber is used for the implementation of the
absorbing boundary condition over the periodic boundary, spe-
cial care needs to be taken for the update of the electric field
nodes that are tangential to the interface between the absorber
and periodic substrate. Our approach is explained in Fig. 14,
where auxiliary magnetic field nodes are introduced within
the periodic substrate along the interface with the absorber to
ensure that the regular Yee updates for the tangential electric fields
(which employ these nodes) can be carried out. These auxiliary
nodes are easily updated by periodic boundary conditions, using
magnetic field values within the unit cell. Hence, their presence
does not add any significant computational cost. Note that at this
region of the substrate, which is beyond the boundaries of the
simulated single unit cell, no other nodes (except for these aux-
iliary ones) need to be updated.

Subsequently, the sine–cosine-based array-scanning analysis
of the structure of [18] is revisited. The structure is considered as
a 1-D periodic geometry in the x-direction. With one unit cell
in that direction, terminated in sine–cosine periodic boundary condi-
tions within the substrate and the aforementioned PML,
the scattering parameters of five unit cells in the y-direction
are computed. One unit cell, without air blocks, is added
before and after these five cells, to provide space for the exci-
tion and probe points, giving rise to a domain of seven unit
cells in total, terminated at a PML as well. The Yee cell size
is 0.5 mm \times 0.5 mm \times 0.318 mm, and hence, a single unit cell
of the structure contains 28 \times 28 \times 30 cells. A 2–10-GHz mod-
uated Gaussian hard source excitation is applied 14 Yee cells
from the first set of air blocks in the propagation direction and
the vertical (E_z) electric field is probed one cell beneath the mi-
crostrip at the two boundaries of the perforated substrate. The
time step is \Delta t = 0.602 \text{ ps} and 16384 time steps are run.

The scattering parameters are then computed with \( N = 16k_{\text{inc}} \)
points that take a total simulation time of 863 s. For comparison,
a finite structure with seven cells in the x-direction is also mod-
eled in 4107 s. The results of the two methods are in good agree-
ment, as shown in Fig. 15, and are corroborated by the theoret-
ical and experimental results of [18]. This agreement can also be
observed in the time domain. To that end, Fig. 16 presents the
time-domain waveform of the transmitted vertical (E_z) elec-
tric field at the output of the simulated five unit cell geometry
(one cell beneath the microstrip), as determined by the sine–cos-
cine-based array-scanning method and the corresponding finite
structure simulation.

Finally, to confirm that the spurious periodic reproduction of the
microstrip line (observed in Fig. 12) is avoided, the Euclidean norm of the x-component of the electric field is sampled
one cell below the air–substrate interface and plotted in Fig. 17.
Note that, in this case, the plane of the air–substrate interface is
not terminated in periodic boundary conditions; therefore, the
values of the field beyond the limits of one unit cell in the trans-
verse direction at the interface are not determined. Hence, \( E_x \) is
now sampled just one cell below this plane within the periodic

Fig. 12. Euclidean norm of the x-component of the electric field on the air–substrate interface of: a microstrip line over a unit cell of the periodic substrate of Fig. 10 terminated in periodic boundary conditions; a finite structure consisting of unit cells of the same periodic substrate, with microstrip lines printed on each one of these cells. The position of the microstrip lines in this finite structure is also shown.

Fig. 13. Combination of periodic and absorbing boundary conditions with array scanning ensures that the original structure can be simulated through the reduced computational domain.

Fig. 14. Update scheme for the tangential electric field nodes that are tangential to the interface between the PML absorber and periodic substrate.
Fig. 15. Scattering parameters of the electromagnetic-bandgap substrate microstrip line of Fig. 10, calculated by the sine–cosine-based array-scanning technique (with \( N = 16 \) \( k_x \) points) and a finite structure simulation, with seven cells in the \( x \)-direction. (a) \( S_{11} \), (b) \( S_{21} \).

Fig. 16. Time-domain waveform of the transmitted vertical \( E_z \) electric field at the output of the simulated five unit cell structure of the electromagnetic-bandgap substrate microstrip line of Fig. 10 (one cell beneath the microstrip), as determined by the sine–cosine-based array-scanning method (with \( N = 16 \) \( k_x \) points) and a finite structure simulation, with seven cells in the \( x \)-direction.

Fig. 17. Euclidean norm of the \( x \)-component of the electric field one Yee cell below the air–substrate interface of: a microstrip line over a unit cell of the periodic substrate of Fig. 10 terminated in periodic boundary conditions within the substrate and an absorber from the air–substrate interface on. A finite structure consisting of seven unit cells of the same periodic substrate, with microstrip lines printed on the center cell. The position of the microstrip line is also shown.

V. CONCLUSION

The sine–cosine method, which enables the FDTD modeling of periodic structures by simulating a single unit cell, was combined with the array scanning technique. Thus, a fast, yet accurate, approach for the time-domain modeling of driven periodic structures was formulated and its potential to achieve significant savings in both computation time and memory was demonstrated. In addition, the application of this method to the particularly interesting class of geometries, consisting of metallic strips printed on periodic substrates, has been discussed. It has been shown that FDTD allows for the stable localized application of periodic boundary conditions and their combination with absorbing boundary conditions to form composite periodic/absorbing boundaries. This result is extremely important for the efficient treatment of periodic structure-based multilayer circuit and antenna problems since it shows that it is possible to restrict the application of Floquet conditions in periodic layers only.

There is one key difference between this approach and its existing alternative, which is modeling a sufficiently large finite number of unit cells. In the former, convergence depends on the number of simulated wave vectors within the Brillouin zone. However, these wave-vector simulations are independent from each other and can be readily performed in parallel at the same time. In addition, their memory cost remains constant without scaling with the number of wave vectors. In the latter, on the other hand, convergence depends on the number of unit cells employed. Accordingly, the memory and execution time of the associated problem grows as more cells are added. While structures with weak coupling between unit cells may converge fast, others (artificial dielectrics, such as the first example of this paper) may need several unit cells before finally converging. For those cases, the method presented here can be particularly useful.
REFERENCES


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